Mihir has stayed up until 4 A.M. trying to solve these Ted-Ed riddles. Help him solve the riddles so he can finally go to sleep before he gets caught.

Sneaky Shreyas walks into China First and pays for \$30 dollars worth of food with a \$100 bill. However the cashier, Justin, doesn't have change for a \$100 dollar bill so he goes to Prabhas's Fish store to break the bill into 10 \$10 bills. Justin then gives adequate change to Shreyas, and Shreyas leaves the store. Ten minutes later, Prabhas walks in mad because the \$100 bill was counterfeit. Prabhas takes \$100 from Justin. Not including the value of the food, let A be the dollar amount of money Justin lost.

Dylan finds 9 identical pencils in his hair, but he can't figure out which one is his. He knows that his pencil weight slightly more than the other pencils, so he gets out his handy balancing scale. Let B be the minimum number of weight comparisons that Dylan needs to make to guarantee that he finds his pencil.

Find A + B.

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Justin chooses a random real number x from the interval  $[0, 2\pi]$ . What is the probability that exactly two of the following statements are true? (Assume that if one or both sides of the inequality are undefined, then the statement is false for that value of x.)

 $A = \cos(2x) + 2\sin^{2}(x) > \sin(x) + 1$   $B = \sec(x) > \csc(x)$  $C = \sin(2x) + 1 > \sin(x) + 2\cos(x)$ 

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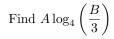
 $X = \text{The eccentricity of the equation defined by the parameters } x = t^2 + t + 1 \text{ and } y = t^2 - t + 1$  $Y = \text{The eccentricity of the equation defined by the parameters } x = \sqrt{3} \left(\frac{1-t^2}{1+t^2}\right) \text{ and } y = \frac{2t}{1+t^2}$ 

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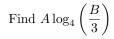
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- A = Find the geometric mean of the arithmetic mean and the harmonic mean of 529 and 729
- B = The minimum value of the function  $f(x) = x^2 + \frac{1}{x}$  for positive values of x (Hint: Split the 1/x term)



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Rohan's vector **r** has magnitude  $4\sqrt{2}$  and Vishnav's vector **v** has magnitude  $7\sqrt{2}$ . The dot product of the two vectors is equal to the magnitude of their cross-product. Both vectors start at the origin and the angle between the vectors is acute.

- A = The distance between the endpoints of the two vectors
- B = The area of the triangle bounded by the two vectors and the line connecting their endpoints
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- A = The probability that  $7^m + 7^n$  is divisible by 5, given that m and n are positive integers between 1 and 100 inclusive.
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- C = The area of the intersection between the graphs  $r = \cos \theta$  and  $r = \sin \theta$

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At 3 pm on Monday, Dylan has 1 pencil behind his ear. Each day, the number of pencils behind his ear, x, changes at 3 pm. On the *n*th day after this Monday at 3 pm, Dylan randomly chooses to either add n pencils to his ear, add 9x + n pencils to his ear, or add  $10^{\lfloor \log(x)+1 \rfloor} \cdot n$  pencils to his ear. After 3pm on Day 6, what is the probability that Dylan has over one hundred thousand pencils behind his ear?

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Let:

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Dylan gets bored of precalculus halfway through the year, so he decides to learn calculus. Help him solve the following differentiation questions.

The function 
$$f'(x)$$
 is defined as  $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ 

 $A = f'(x) \text{ where } f(x) = x^2$  $B = f'(x) \text{ where } f(x) = \sqrt{x}$ 

The first 3 terms of a Maclaurin Series for a function f(x) is defined as  $\frac{f(0)}{0!} + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!}$  where  $f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$ 

C = The first 3 terms of the Maclaurin series for  $f(x) = x^2 - 3x + 4$ 

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Dylan still needs more help with calculus. Now help him solve the following integration questions.

The value of the integral  $\int_{n}^{k} f(x)dx$  is defined as (Area bounded by f(x), the lines x = n and x = k that is above the x-axis) – (Area bounded by f(x), the lines x = n and x = k that is below the x-axis)

$$A = \int_{-1}^{2} (-|x|+1) dx$$
  

$$B = \int_{0}^{1.5} \sqrt{9 - 4x^2} dx$$
  

$$C = \int_{-\frac{\pi}{4}}^{\frac{7\pi}{4}} (\sin x + \cos x)^2 dx$$



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Farzan is studying electrical engineering and finds that the equivalent resistance for a set of n resistors  $R_0, R_1, \dots, R_n$  is the reciprocal of the sum of their reciprocals. Farzan, however, has an infinite number of resistors! (Equivalent resistance is calculated in the same way, except the sum of the reciprocals is now an infinite series.)

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Triangle ABC has side lengths of tan(A), tan(B), tan(C) opposite of angles  $\angle A, \angle B$ , and  $\angle C$  respectively. Let P be the area between the incircle and the circumcircle of triangle ABC.

Triangle DEF has one side of length 12 with opposite angle of 60°. Let Q be the length of the largest latus rectum of a parabola given that the vertex and the focus of the parabola are on the circumcircle of  $\triangle DEF$ .

If a triangle has sides of length P and Q with an included angle of  $30^{\circ}$ , what is its area?

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